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SYNTHESIS OF THERMOSTATING DEVICES. III. MINIMIZATION OF THE ERROR OF THERMOSTATING

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The article suggests a method of choosing the design parameters for a heater-type thermostat. Approximate formulas are obtained for determining the dynamic error in on-off control, and the problem of its minimization is dealt with.

Statement of the Problem. Synthesis of the optimal design of a thermostating device presupposes the choice of design and regime parameters ensuring the minimum of some target function which also includes quality indices of the device to be designed. Among these indices may be the static and dynamic errors of thermostating, the time required to attain the operating regime, characteristics of weight and overall dimensions, etc. The problem of choosing the optimal design and regime parameters can be solved on the basis of various mathematical models of thermostats dealt with in [1]. If the mathematical model is not suitable for establishing a fairly simple analytical dependence of the target function on the variable parameters, then for the sake of optimal choice of parameters numerical methods of optimization have to be used or the problem has to be solved by the method of nonformalized scanning of the variants. Often it is found that both approaches are ineffective unless the nature of the dependence of the quality indices on the variable parameters had been previously investigated and preliminary evaluations of the ranges had been obtained within which the optimal values of the parameters lie. It is advisable to carry out such a preliminary analysis with the aid of the simplest mathematical models that yield analytical expressions for the investigated quality indices.

The present article examines the problem of minimizing the static and dynamic errors of thermostating by means of optimal choice of parameters of a heater-type thermostat on the basis of models with lumped parameters. As elements the simplest model contains the object of thermostating 1, the chamber 2 with controlled power, and sensor 3 mounted on the chamber. We denote the temperature of these elements by t_1, t_2, and t_3, and we write the system of equations of thermal balance [1]:

$$P_1 = C_1 \frac{dt_1}{d\tau} + \sigma_{12}(t_1 - t_2) + \sigma_{1c}(t_1 - t_c), \quad (1)$$

$$P_2 = C_2 \frac{dt_2}{d\tau} + \sigma_{12}(t_2 - t_1) + \sigma_{23}(t_2 - t_3) + \sigma_{2c}(t_2 - t_c), \quad (2)$$

$$0 = C_3 \frac{dt_3}{d\tau} + \sigma_{23}(t_3 - t_2) + \sigma_{3c}(t_3 - t_c). \quad (3)$$

The power P_2 of the final control element situated on the chamber depends on the temperature of the sensor t_3. Confining ourselves to considering on-off control, we write this dependence in the form

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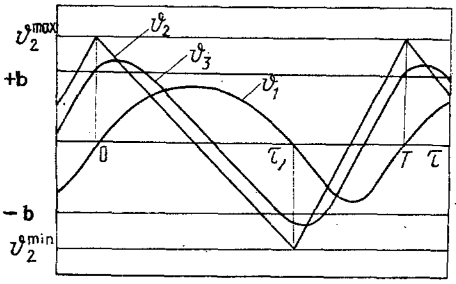


Fig. 1. Nature of the change of overheating of the thermostat elements in the regime of free oscillations.

$$P_2 = P_3(\vartheta_3) = \begin{cases} P_{\max} \text{ for } \vartheta_3 < -b \text{ or } \begin{cases} -b \leqslant \vartheta_3 \leqslant b, \\ \dfrac{d\vartheta_3}{d\tau} > 0, \end{cases} \\ 0 \text{ for } \vartheta_3 > b \text{ or } \begin{cases} -b \leqslant \vartheta_3 \leqslant b, \\ \dfrac{d\vartheta_3}{d\tau} < 0, \end{cases} \end{cases} \tag{4}$$

where $\vartheta_3 = t_3 - t_{t.s}$ is the deviation of the temperature of the sensor from its setting $t_{t.s}$.

When a thermostating device is being designed, the values P_1, C_1, σ_{1c} characterizing the object are assumed to be specified. The object of the present work is to substantiate the choice of the parameters σ_{12}, σ_{2c}, σ_{23}, σ_{3c}, C_2, C_3, P_2, b on the basis of the mathematical model (1)-(4). To choose these parameters, we will consider separately their effect on the static and dynamic components of the error of thermostating.

Static Error. By static error Δ_{st} we mean the difference between the maximal and minimal values of the temperature of the object t_1 established upon change of temperature of the medium from t_c^{min} to t_c^{max}. The expression for Δ_{st} has the form [2]

$$\Delta_{st} = (1 - \varepsilon)(t_c^{\max} - t_c^{\min}),\ \varepsilon = \frac{1 + \sigma_{3c}/\sigma_{23}}{1 + \sigma_{1c}/\sigma_{12}}. \tag{5}$$

From among the parameters subject to choice, relation (5) contains the thermal conductivities σ_{12}, σ_{3c}, σ_{23} which have to be chosen such that the static error is minimal. It can be seen from (5) that this is attained with $\varepsilon = 1$, i.e., with

$$\sigma_{1c}/\sigma_{12} = \sigma_{3c}/\sigma_{23}. \tag{6}$$

The thermal conductivity σ_{12} depends on the method of fastening the object inside the chamber and on the conditions of heat exchange between the object and the chamber. The range of possible change of the value of σ_{12} is determined at the stage when the basic model is selected. The thermal conductivities σ_{23} and σ_{3c} are characteristics of the temperature sensor that is used and of the method of mounting it in the chamber. These conductivities for each actual type of sensor and for certain conditions of mounting it are approximately known.

The sequence of choosing the conductivities ensuring a minimal value of Δ_{st} is best arranged in the following manner. First the actual value of σ_{12} is chosen in the first approximation from the interval $[\sigma_{12}^{min}, \sigma_{12}^{max}]$ determined in the selection of the basic model. Then, in accordance with relation (6), we choose the temperature sensor and the conditions of mounting it. Since the ratio σ_{3c}/σ_{23} for different conditions changes discretely, we need a more accurate value of σ_{12} to fulfill (6) accurately. In view of the inevitable differences between the real conductivities in a finished structure and the assumed calculated values, and also on account of the fact that in some cases the ranges of possible change of these conductivities cannot ensure that $\varepsilon = 1$, not in any combination, compensation of the static error [2] is envisaged in thermostats.

Dynamic Error. By dynamic error we mean the amplitude of the oscillations A_1 of the temperature of the object in operating regime with constant ambient temperature. To obtain the expression for the dynamic error, we transform the system (1)-(3). For that purpose we determine the mean temperature of the object $\bar{t}_1$ and of the chamber $\bar{t}_2$, and also the mean power of the heater $\bar{P}$, as the solution of the steady-state problem corresponding to (1)-(3) with constant temperature of the sensor $\bar{t}_3$ equal to the temperature of its setting $t_{t.s}$:

$$\overline{t}_1 = \frac{\sigma_{12}\overline{t}_2 + \sigma_{1c}t_c + P_1}{\sigma_{12} + \sigma_{1c}}, \quad \overline{t}_2 = \frac{(\sigma_{23} + \sigma_{3c})t_{t.s} - \sigma_{3c}t_c}{\sigma_{23}},$$

$$\overline{P} = (\sigma_{12} + \sigma_{23} + \sigma_{2c})\overline{t}_2 - \sigma_{12}\overline{t}_1 - \sigma_{23}t_{t.s} - \sigma_{2c}t_c. \tag{7}$$

We will introduce into the consideration the magnitudes ϑ_1, ϑ_2, ϑ_3 which are the deviations of the temperatures of the respective elements from their mean values: $\vartheta_i(\tau) = t_i(\tau) - \overline{t}_i$, i = 1, 2, 3. In Eq. (2) for the chamber we replace the thermal fluxes from the chamber to the object, the sensor, and the environment by their mean values $\sigma_{12}(\overline{t}_2 - \overline{t}_1)$, $\sigma_{23}(t_2 - t_{t.s})$, $\sigma_{2c}(\overline{t}_2 - t_c)$; below we will discuss this assumption. Then the system (1)-(3), written in deviations, assumes the form

$$C_1\frac{d\vartheta_1}{d\tau} + (\sigma_{12} + \sigma_{1c})\vartheta_1 - \sigma_{12}\vartheta_2 = 0, \tag{8}$$

$$C_2\frac{d\vartheta_2}{d\tau} = \begin{cases} -\overline{P}, & 0 \leqslant \tau < \tau_1, \\ \Delta P = P^{\max} - \overline{P}, & \tau_1 \leqslant \tau < T, \end{cases} \tag{9}$$

$$C_3\frac{d\vartheta_3}{d\tau} + (\sigma_{23} + \sigma_{3c})\vartheta_3 - \sigma_{23}\vartheta_2 = 0. \tag{10}$$

The solution of the system (8)-(10) must satisfy the conditions of periodicity

$$\vartheta_i(0) = \vartheta_i(T), \; i = 1, 2, 3, \tag{11}$$

and the conditions of continuity at the instants when the heater is switched over

$$\vartheta_i(\tau_1 - 0) = \vartheta_i(\tau_1 + 0), \; i = 1, 2, 3. \tag{12}$$

With the adopted assumptions, the temperature of the chamber in the regime of free oscillations changes linearly (Fig. 1), and the solution of Eq. (9) can be written in the form

$$\vartheta_2(\tau) = \begin{cases} \vartheta_2^{\max} - \dfrac{\overline{P}}{C_2}\tau, & 0 \leqslant \tau < \tau_1, \\ \vartheta_2^{\min} + \dfrac{\Delta P}{C_2}(\tau - \tau_1), & \tau_1 \leqslant \tau < T, \end{cases} \tag{13}$$

where $\vartheta_2^{\max}$, $\vartheta_2^{\min}$ are the maximal and minimal values, respectively, of the deviation $\vartheta_2(\tau)$ at the instants $\tau = 0$, $\tau = \tau_1$; these values have to be determined later.

The general solution of Eq. (10) for the sensor has the form [3]

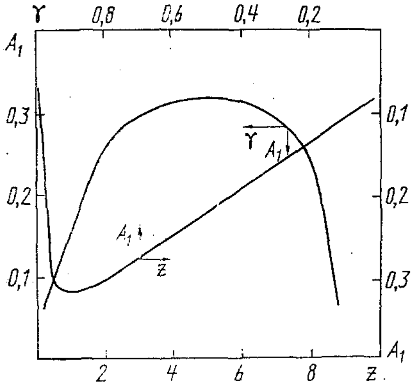
$$\vartheta_3(\tau) = \vartheta_3(\tau^*)\exp(-m_3\tau) + \exp(-m_3\tau)\int_{\tau^*}^{\tau}\frac{\sigma_{23}}{C_3}\vartheta_2(\tau)\exp(m_3\tau)\,d\tau, \tag{14}$$

where $m_3 = (\sigma_{23} + \sigma_{3c})/C_3$ is the cooling rate of the sensor; τ^* is the initial instant of the time interval under consideration; $\vartheta_2(\tau)$ is the dependence determined by relations (13).

Bearing in mind that at the instants of switching the heater off ($\tau = 0$) and on ($\tau = \tau_1$), the sensor is overheated by $\vartheta_3(0) = b$ and $\vartheta_3(\tau_1) = -b$, we can easily obtain from (14) the solutions that describe the change of overheating of the sensor $\vartheta_3(\tau)$ in the cooling and heating sections of the chamber. The nature of the change of overheating is shown in Fig. 1. We adopt the assumption that the sensor has low thermal inertia, i.e., its time constant $1/m_3$ is substantially smaller than the cooling time τ_1 and the heating time $\tau_2 = T - \tau_1$ of the chamber. Then we may neglect the terms with exponential factors in the solution for $\vartheta_3(\tau)$ and assume that upon approach to the instants τ_1 and T the overheating of the sensor changes linearly (see Fig. 1), i.e., a regular regime of the second kind begins. On the sections of linear change of temperature the expressions for $\vartheta_3(\tau)$ have the form:

$$\vartheta_3(\tau) = \beta\left[\vartheta_2(\tau) + \frac{\overline{P}}{C_2m_3}\right], \; \tau < \tau_1, \tag{15}$$

$$\vartheta_3(\tau) = \beta\left[\vartheta_2(\tau) - \frac{\Delta P}{C_2m_3}\right], \; \beta = \sigma_{23}/(\sigma_{23} + \sigma_{3c}), \; \tau_1 < \tau < T. \tag{16}$$



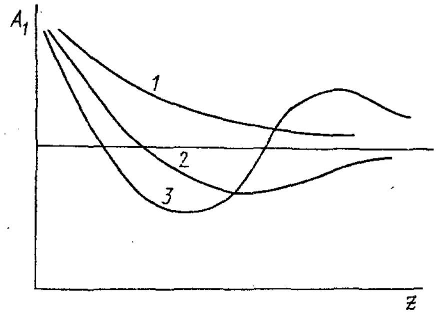


Fig. 2 Fig. 3

Fig. 2. Dependence of the amplitude of the free oscillations calculated by (22) on the parameter z and the duty factor γ. A_1, °K; z, sec/°K.

Fig. 3. Nature of the dependence of the amplitude of the free oscillations on the parameter z in accordance with (23): 1) $X < 1/m_3$; 2) $X = 1/m_3$; 3) $X > 1/m_3$ ($X = 2\pi\gamma(1-\gamma)/m_1$).

It follows from the conditions (11), (12) that $\vartheta_3(\tau_1 - 0) = -b$, $\vartheta_3(T) = b$. Substituting these magnitudes into (15), (16) we can easily resolve them in respect to $\vartheta_2^{\min}$, $\vartheta_2^{\max}$ and find the expression for the amplitude of the temperature oscillations of the chamber:

$$A_2 = \vartheta_2^{\max} - \vartheta_2^{\min} = 2b\frac{(\sigma_{23}+\sigma_{3c})}{\sigma_{23}} + \frac{P^{\max}C_3}{C_2(\sigma_{23}+\sigma_{3c})}\,. \quad (17)$$

Formula (17) has a simple physical meaning: the first term characterizes the effect of the zone of ambiguity of the regulator, and the second term characterizes the temperature lag of the sensor behind the temperature of the chamber in consequence of its thermal inertia.

It follows from (13) that the times of cooling τ_1 and of heating τ_2 the chamber are correlated with the oscillation amplitude A_2 by the ratios

$$\tau_1 = \frac{C_2}{\bar{P}}(\vartheta_2^{\max} - \vartheta_2^{\min}), \quad \tau_2 = \frac{C_2}{\Delta P}(\vartheta_2^{\max} - \vartheta_2^{\min}). \quad (18)$$

Then the period of free oscillations T with a view to (17) is equal to

$$T = \frac{C_2 P^{\max}}{\bar{P}\Delta P} A_2. \quad (19)$$

We will now go over to determining the oscillation amplitude of the temperature of the object. For that we substitute expression (13) for $\vartheta_2(\tau)$ into Eq. (8). The solution of the equation of heat balance for the object has a form analogous to (14). To obtain simple approximate dependences, we will consider the case of an object having great thermal inertia, i.e., its time constant $1/m_1 = C_1/(\sigma_{12} + \sigma_{1c})$ exceeds considerably the period of free oscillations. Then, expanding the exponential factors in the solution for $\vartheta_1(\tau)$ into a Taylor series and confining ourselves to the terms containing the small parameter $m_1\tau$ in the first degree, we obtain the expression for overheating of the object:

$$\vartheta_1(\tau) = \alpha\left[\bar{\vartheta}_2 + m_1\tau\left(\frac{A_2}{2} - \frac{\bar{P}}{C_2}\frac{\tau}{2}\right)\right], \quad 0 \leqslant \tau < \tau_1, \quad (20)$$

$$\vartheta_1(\tau) = \alpha\left[\bar{\vartheta}_2 + m_1(\tau-\tau_1)\left(-\frac{A_2}{2} + \frac{\Delta P}{C_2}\frac{(\tau-\tau_1)}{2}\right)\right], \quad \tau_1 \leqslant \tau < T, \quad (21)$$

where

$$\bar{\vartheta}_2 = \frac{1}{2}(\vartheta_2^{\max} + \vartheta_2^{\min}), \quad \alpha = \sigma_{12}/(\sigma_{12} + \sigma_{1c}).$$

The functions $\vartheta_1(\tau)$, determined by the relations (20), (21), have the extrema $\vartheta_1^{\max}$ and $\vartheta_1^{\min}$ at the instants $\tau = \tau_1/2$ and $\tau = \tau_1 + \tau_2/2$, respectively. Taking (18) into account, we find

$$\vartheta_1^{\max} = \alpha\left(\bar{\vartheta}_2 + \frac{m_1\tau_1}{8} A_2\right), \quad \vartheta_1^{\min} = \alpha\left(\bar{\vartheta}_2 - \frac{m_1\tau_2}{8} A_2\right),$$

and the amplitude of the temperature oscillations of the object A_1 is

$$A_1 = \frac{\sigma_{12}}{8C_1} A_2 T. \tag{22}$$

If the assumption that the object has great thermal inertia is incorrect, we can obtain a simple evaluation of the magnitude of the oscillation amplitude A_1 by replacing the linear change of temperature of the chamber $\vartheta_2(\tau)$ by harmonic oscillations with the same amplitude A_2 and period T. In that case the amplitude of the temperature oscillations of the object is described by the expression

$$A_1 = A_2\left[1 + \left(\frac{2\pi}{T} \frac{C_1}{(\sigma_{12} + \sigma_{1c})}\right)^2\right]^{-\frac{1}{2}}, \tag{23}$$

in which A_2 and T are determined in accordance with (17), (19).

Thus, the obtained simple analytical dependences for the parameters of the regime of free oscillations of the thermostat make it possible to analyze the extremal properties of the dynamic error.

We emphasize the assumptions adopted in deriving the approximate dependences. Firstly, it was assumed that the changes of the thermal fluxes from the chamber to the object, to the sensor, and to the environment compared with the sum of their mean values are:

$$[\sigma_{12}(t_2 - t_1) + \sigma_{23}(t_2 - t_3) + \sigma_{2c}(t_2 - t_c) - \bar{P}]/\bar{P} = \delta_2(\tau) \ll 1.$$

Secondly, the time constant of the sensor has to be small compared with the times of cooling and heating of the chamber:

$$C_3/[(\sigma_{23} + \sigma_{3c})\min(\tau_1, \tau_2)] = \delta_3 \ll 1.$$

These two assumptions were used in deriving expressions (17), (19) for A_2 and T. Expression (22) for the amplitude A_1 is correct with the additional assumption that the time constant of the object is large:

$$T(\sigma_{12} + \sigma_{1c})/C_1 = \delta_1 \ll 1.$$

The error of the calculation by the approximate dependences was evaluated by comparing it with the results of the rigorous solution of the problem (1)-(4) obtained by the method of "fitting" the accurate analytical solutions for the sections of heating and cooling [4]. With $\delta_1 \le 0.5$, $\delta_2 \le 0.2$, $\delta_3 \le 0.3$ the relative error of calculating the amplitude of the free oscillations by (22) does not exceed 15%, and with $\delta_1 > 0.05$, $\delta_2 \le 0.2$, $\delta_3 \le 0.3$ and $0.2 \le \tau_1/\tau_2 \le 5$ the error of calculation by (23) does not exceed 25%. An analysis of the parameters of real thermostat designs showed that the formulated constraints are fulfilled for a broad class of objects.

To investigate the extremal properties of the dynamic error, we rewrite formula (22) in the form

$$A_1 = \frac{\sigma_{12}}{8C_1} \frac{[\varphi(z, \gamma)]^2}{z(1 - \gamma)}, \tag{24}$$

where

$$\varphi(z, \gamma) = 2b\frac{(\sigma_{23} + \sigma_{3c})}{\sigma_{23}} z + \frac{C_3}{(\sigma_{23} + \sigma_{3c})\gamma},$$

$$z = C_2/\bar{P}, \quad \gamma = \bar{P}/P^{\max}, \quad 0 < \gamma < 1.$$

It follows from (24) that the dynamic error increases monotonically with an increase of the parameters σ_{12}, b, C_3, and decreases monotonically with an increase of σ_{23}. The dependence of $A_1(z, \gamma)$ on the duty factor γ and on the ratio $z = C_2/\bar{P}$ is of nonmonotonic nature. The shape of these dependences for the actual example considered below is shown in Fig. 2. From the necessary conditions of existence of an extremum

$$\frac{\partial A_1}{\partial z} = 0, \quad \frac{\partial A_1}{\partial \gamma} = 0$$

we can easily obtain the optimal values z_{opt} and γ_{opt} corresponding to the minimum of the dynamic error:

$$\gamma_{opt} = 0{,}5,\ z_{opt} = \frac{1}{b}\ \frac{C_3\sigma_{23}}{(\sigma_{23}+\sigma_{3c})^2}, \qquad (25)$$

$$A_1^{min} = A_1(z_{opt}, \gamma_{opt}) = 4b\ \frac{\sigma_{12}C_3}{C_1\sigma_{23}}.$$

We note that the optimal value of the duty factor $\gamma = 0.5$ is in agreement with the conclusions arrived at by other authors, in particular by Ingberman et al. [2], and the recommendation for the choice of heat capacity of the chamber C_2 is a new result.

An analogous investigation of the optimal values of z and γ was carried out for the dependence (23) but the obtained formulas have a more cumbersome form. The qualitative form of the dependence $A_i(z)$ plotted by formula (23) is shown in Fig. 3; with some combinations of parameters, the parameter z may remain without an extremum.

The parameters z and γ do not affect the static error Δ_{st}; the power of the final control element P^{max} and the heat capacity of the chamber C_2 in the synthesis of the thermostat may therefore be chosen from the conditions of minimum of the dynamic error (25). The values of σ_{12} and the ratio σ_{23}/σ_{3c} are determined in accordance with the method adopted above which is based on the analysis of the magnitude of the static error. The values b and C_3, on which the dynamic error depends monotonically, have to be chosen such that the error A_1, determined by (22) or (23), does not exceed the permissible values. We note that after a preliminary choice of the thermostat parameters on the basis of the examined model with lumped parameters it is advisable to carry out more accurate calculations according to more complete models [1]. Then it is possible to pose the question of nonlinear programming and to realize it with the numerical method of optimization because the results obtained above make it possible to evaluate the nature of the dependence of the target function on the variable parameters and to evaluate the region where the optimal values lie.

Example of the Choice of Thermostat Parameters. We thermostat an object whose heat capacity C_1 is equal to 1.0 J/°K, and the thermal contact with the environment is evaluated by the conductivity $\sigma_{1c} = 0.05$ W/°K. There are no internal heat sources in the object ($P_1 = 0$). In the planned thermostat it is intended to use an on-off regulator with the width of the zone of ambiguity 2b = 1°K and a sensor with the parameters $C_3 = 0.2$ J/°K, $\sigma_{23} = 0.5$ W/°K, $\sigma_{3c} = 0.025$ W/°K. Conductivity between the chamber and the environment is evaluated by the magnitude $\sigma_{2c} = 0.05$ W/°K. On the object the temperature $t_1 = 70$°C has to be maintained while the ambient temperature may change from $t_c^{min} = 15$ to $t_c^{max} = 35$°C.

It is necessary to determine the conductivity between the object and the chamber σ_{12}, the power of the final control element P^{max}, and the heat capacity of the chamber C_2.

We determine the conductivity σ_{12} on the basis of the requirement that there be no static error. Then, on the basis of (6), we obtain $\sigma_{12} = 0.1$ W/°K.

We determine the values of $\bar{P}$ and C_2 on the basis of the requirement of the minimal dynamic error at an ambient temperature of 25°C, with the mean value out of the possible ones. On the basis of the solution of the steady variant of system (1), $\bar{P} = 3.7$ W. The optimal duty factor is $\gamma = 0.5$; this corresponds to the heater power $P^{max} = 7.4$ W. The dynamic error is evaluated according to (22). The dependence of A_1 on the parameter $z = C_2/\bar{P}$ with $\gamma = 0.5$ is shown in Fig. 2. The minimum of A_1 equal to 0.08°K is attained with z = 0.726; this corresponds to the heat capacity of the chamber $C_2 = 2.7$ J/°K.

With the selected values of the parameters σ_{12}, P^{max}, and C_2 the dynamic error changes from 0.08°K (at 25°C) to 0.09°K (at $t_c = t_c^{min}$ or $t = t_c^{max}$).

In the examined example all the three assumptions made in deriving the approximate dependences are correct: $\delta_1 = 0.4$; $\delta_2|_{\tau=0} = 0.1$; $\delta_3 = 0.4$. Consequently, the error of the calculations does not exceed 15%.

NOTATION

t_i, C_i, temperatures and full heat capacities, respectively, of the object (1), the chamber (2), and of the sensor (3); P_1, full power of the heat sources in the object; σ_{1c}, σ_{2c}, σ_{3c}, thermal conductivities of the object, the chamber, and the sensor, respectively, to the environment; σ_{12}, σ_{23}, thermal conductivities between the chamber and the object, between the chamber and the sensor, respectively; b, half the width of the zone of ambiguity of the regulator; ϑ_i (i = 1, 2, 3), deviation of the temperatures of the elements from their mean values $\bar{t}_1$, $\bar{t}_2$, $\bar{t}_3 = t_{t.s}$; $t_{t.s}$, set temperature of the sensor; $\bar{P}$, P^{max}, mean and maximal power, respectively, of the final control element; T, period of free oscillations; τ_1, τ_2, cooling and heating time, respectively, of the chamber; A_1, A_2, amplitudes of the temperature oscillations of the object and of the chamber, respectively; γ, duty factor of the operation of the final control element.

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SIMULATING THE COOLING OF SPIRAL COMPONENTS IN CIRCULATION SYSTEMS FOR GAS COOLING. PART 1. SINGLE PANCAKE COIL

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UDC 536.24:537.312.62

A study has been made of the effects of design and thermal parameters on the temperature patterns and cooling times for pancake coils.

A large superconducting magnet with circulating coolant is frequently built up from two-layer disk sections [1-3]; each layer or pancake is a flat (archimedean) spiral formed out of insulated hollow wire and embedded in epoxide resin. The cooling channels in adjacent pancakes are usually connected in parallel. The pancake coils in equipments may differ in design, conductor length, number of turns, and insulating material and thickness. There is heat transfer through the insulation between turns and between coils, which sometimes has a substantial effect on the cooling. The mode of cooling must be chosen such that no dangerous thermal stresses arise, while the cooling time and coolant consumption are acceptable. It is possible to choose a state meeting these requirements by solving the nonstationary conjugate heat-transfer problem. The term conjugate here incorporates the fact that it is necessary to solve the energy-conservation equations together for all the components in the heat-transfer system (channel walls and flows) [4, 5].

The cooling of a single adiabatic channel has been examined in most detail (with ideal insulation between turns for a spiral). If the thermal parameters and coolant flow rate are constant, one can obtain an analytic solution if the coolant temperature at the inlet changes stepwise [4]. To allow for the change in heat-transfer coefficient along the channel and for the temperature dependence of the thermophysical parameters, one has to use numerical methods such as [6, 7] to solve the problem with general boundary conditions.

It is recommended [8, 9] that the dimensionless parameter $St^* = \alpha \Pi L/(Gc_p)_g$ should be used in distinguishing long channels ($St^* \geq 100$) from short ones ($St^* \leq 10$); long means that the zone of rapid heat transfer is substantially shorter than the channel, so one can use a temperature-step model to calculate the cooling [10], which can be used with a coolant inlet temperature step to estimate the cooling time from $\tau_b = (M_c)_w/(Gc_p)_g$, which follows from the heat-balance equation, and also to determine the pressure drop or coolant flow rate. A formula has been given [9] for the cooling time for a short channel.

An analytic solution can also be obtained [5, 11] for two parallel long channels with thermal interaction; solutions have been obtained for direct-flow and countercurrent forms of coolant motion for constant thermophysical parameters of the coolant wall, infinitely small thermal capacity of the bridge between channels, and inlet coolant temperature steps. In

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